Conjunction Assessment Risk Analysis

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NASA CARA

Overview

Objectives

At the end of this module you should be able to…

- ✦Introduce the RSO Catalog
- ✦Understand the anatomy of a conjunction
- ✦Describe the Conjunction Analysis Process
- ✦Describe Two-line Element Sets
- ✦Understand covariance and uncertainty
- ✦Describe conjunction assessment screenings
- ✦Explain the risk assessment for CA
- ✦Describe the conjunction situation
- ✦Explain the probability of collision algorithm
- ✦Describe practical problems in determining the probability of collision

Outline

- ✦ Introduction to Conjunction Assessment
- ✦ Conjunction Assessment Process
- ✦ Covariance and Uncertainty
- ✦ Probability of Collision
- ✦ Practical Issues with CA
- ✦ CA Risk Assessment and Mitigation

Adapted from information in:

Space Mission Analysis and Design (SMAD), Chapters 3 and 7 by Wertz
Human Spaceflight: Mission Analysis and Design (HSF), Chapter 9 by Boden and Hoffman
Orbital Mechanics by Logsdon
Space Domain Awareness, Chapter 4 by Ziebart, Frueh and Hejduk

Agenda

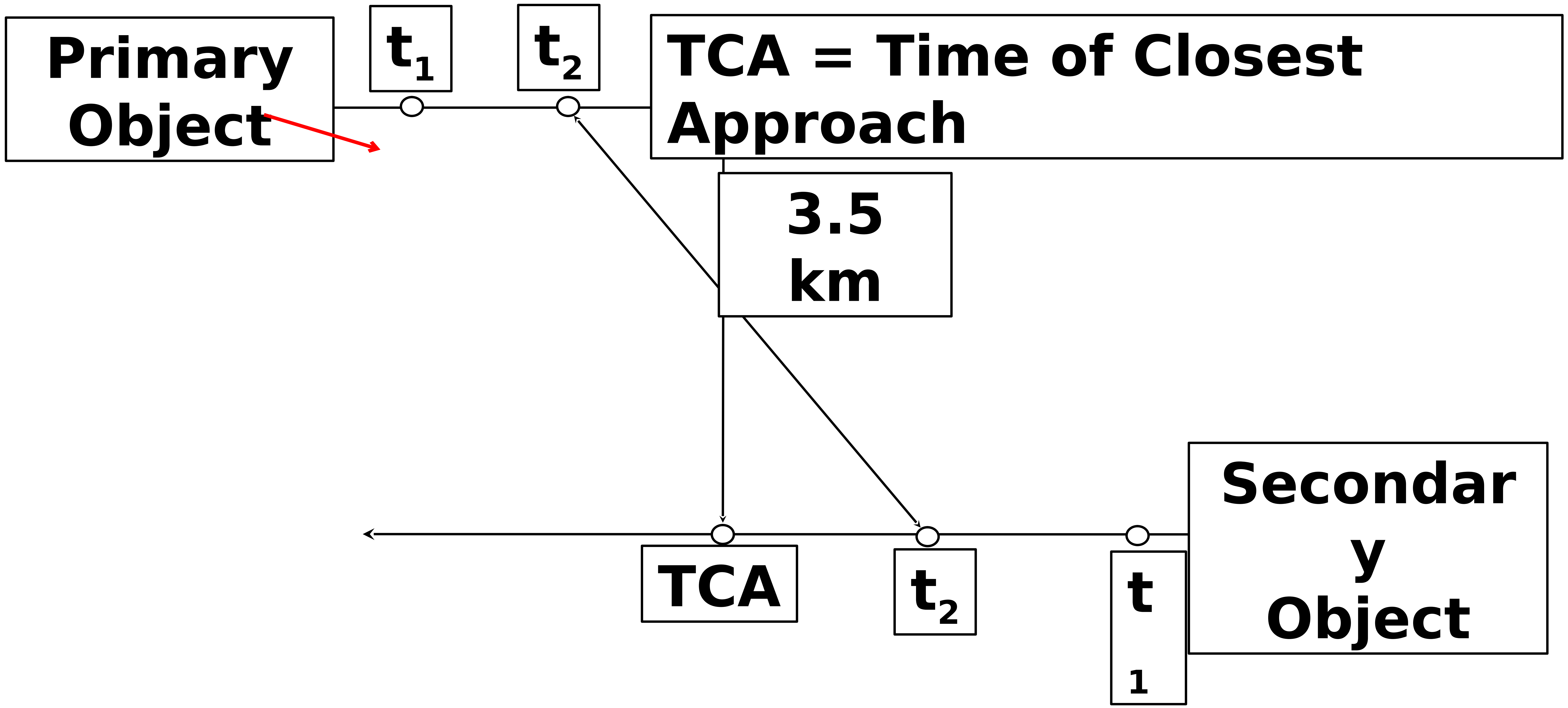
- Conjunction Assessment (CA) terminology and very high-level concepts
- Space catalogue maintenance basics
  - Collecting satellite position data
  - Updating and propagating orbits
- Orbit Determination (OD) uncertainty modeling through covariance
- Probability of collision computation
- Conjunction Assessment screenings
- Conjunction Data Message (CDM) contents

Conjunction Assessment Terms (1 of 6)

- Conjunction Assessment (CA)
  - An iterative process for determining the Time of Closest Approach (TCA) of two tracked orbiting objects or between a tracked orbiting object and a launch vehicle (including spent stages) or payload
    - TCA will be defined shortly
  - Further activities to identify high-interest conjunction events
- Conjunction
  - When the predicted miss distance between two on-orbit objects, or between a launch vehicle and an orbiting object, is less than a specified reporting volume
- Risk Analysis (RA)
  - For a particular conjunction, the determination of the collision likelihood and consequence
  - This information is used to identify high-interest events (HIEs) that may require warnings and conjunction mitigation actions

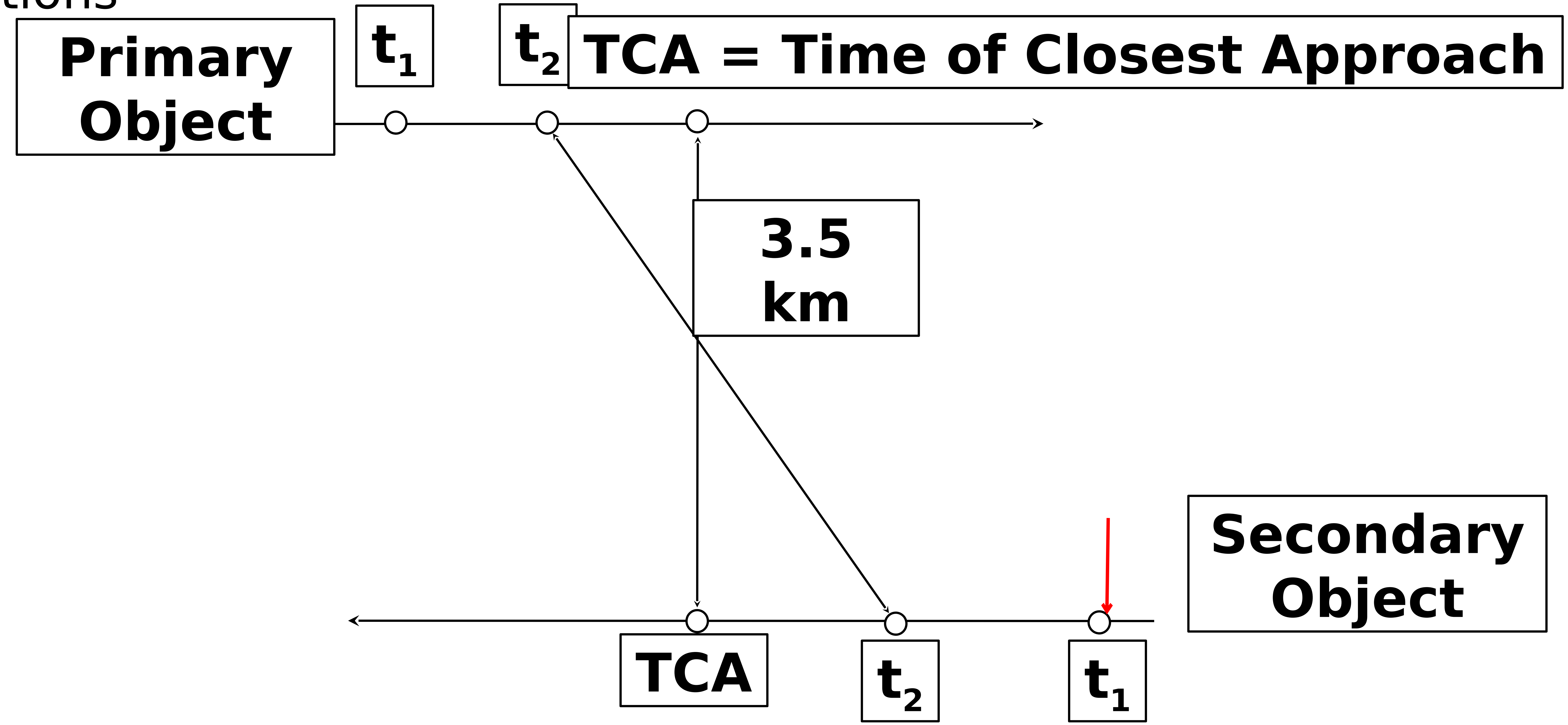
Conjunction Assessment Terms (2 of 6)

- Primary Object
  - The satellite asset, launched object or the ephemeris file that is being screened for potential conjunctions

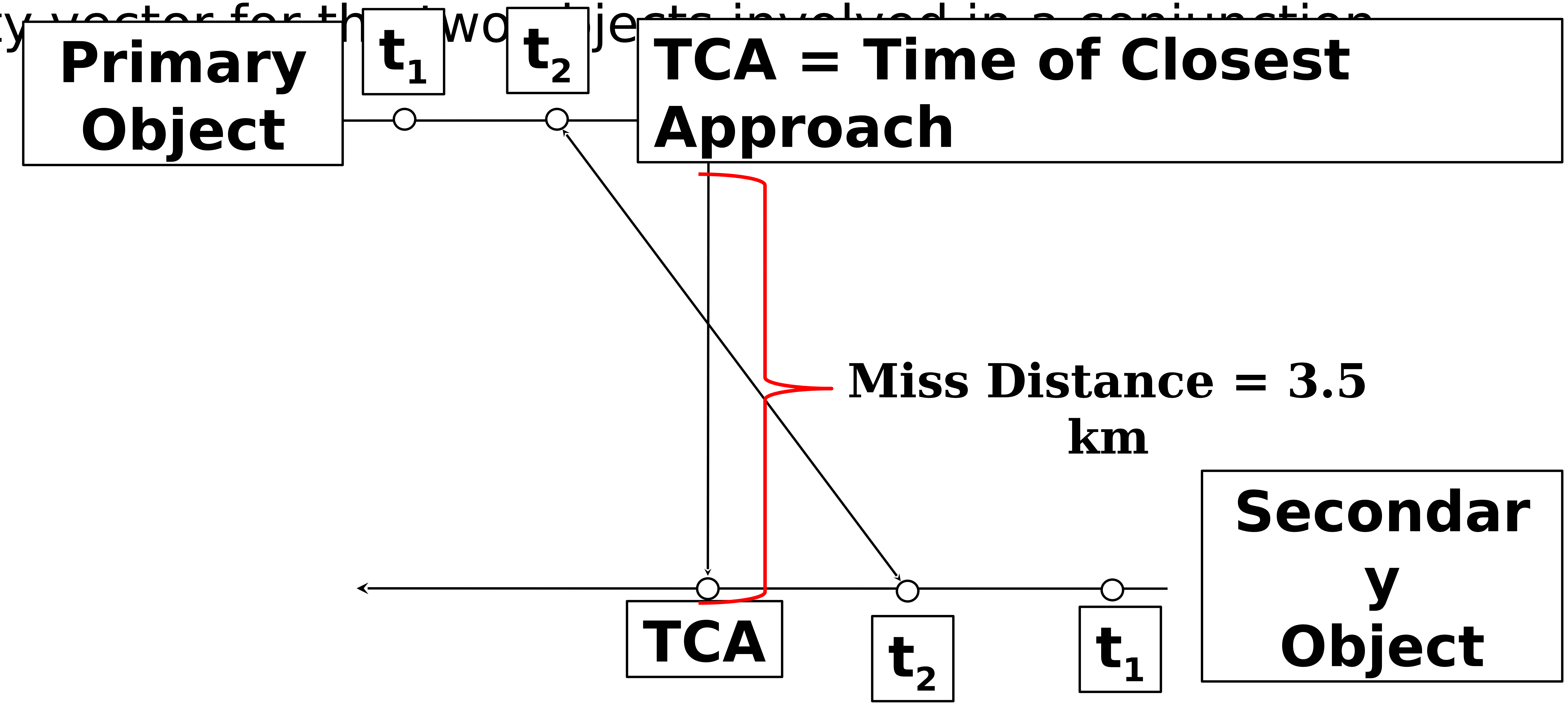


Conjunction Assessment Terms (3 of 6)

- Secondary Object
  - All other satellite objects (examples: payloads, debris, R/B, or analyst satellites) against which the primary object is being screened for potential conjunctions

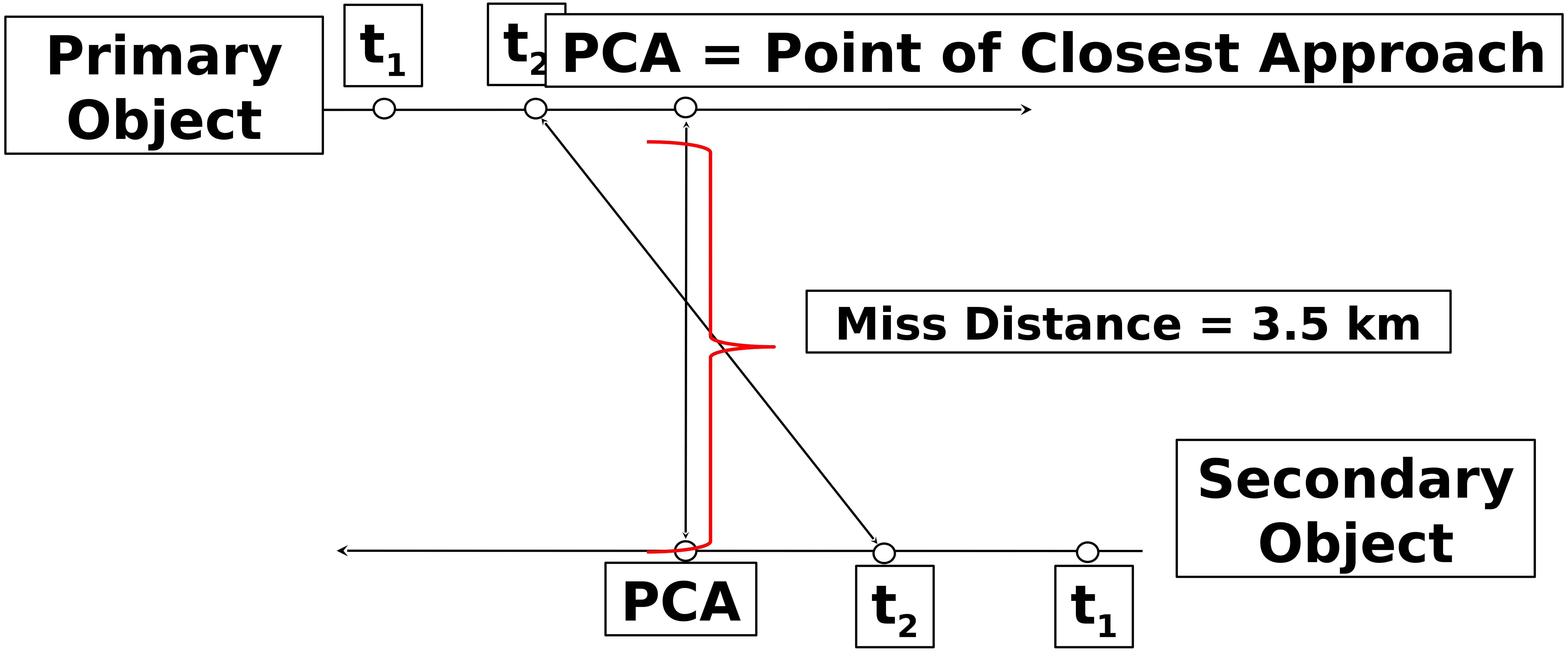


- Time of Closest Approach (TCA)
  - The time at which the distance between the two objects reaches a minimum
    - This occurs when the relative position vector is perpendicular to the relative velocity vector for the two objects involved in a conjunction



Conjunction Assessment Terms (5 of 6)

- Overall Miss Distance
  - The PCA of one object relative to another; i.e., the minimum range, miss distance, or relative position magnitude between two satellites at TCA
    - Can also be expressed by individual three-dimensional component

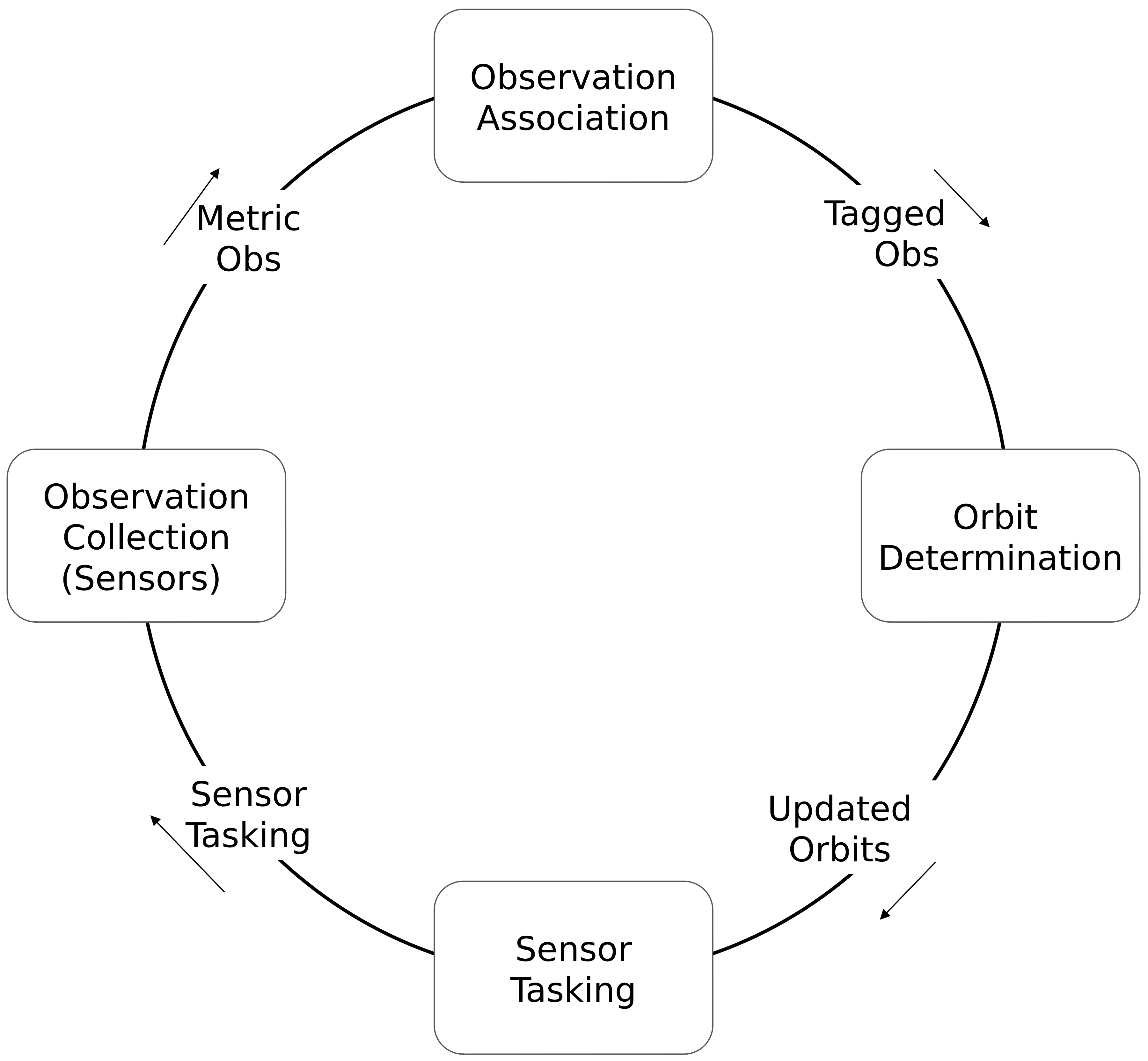


Conjunction Assessment Terms (6 of 6)

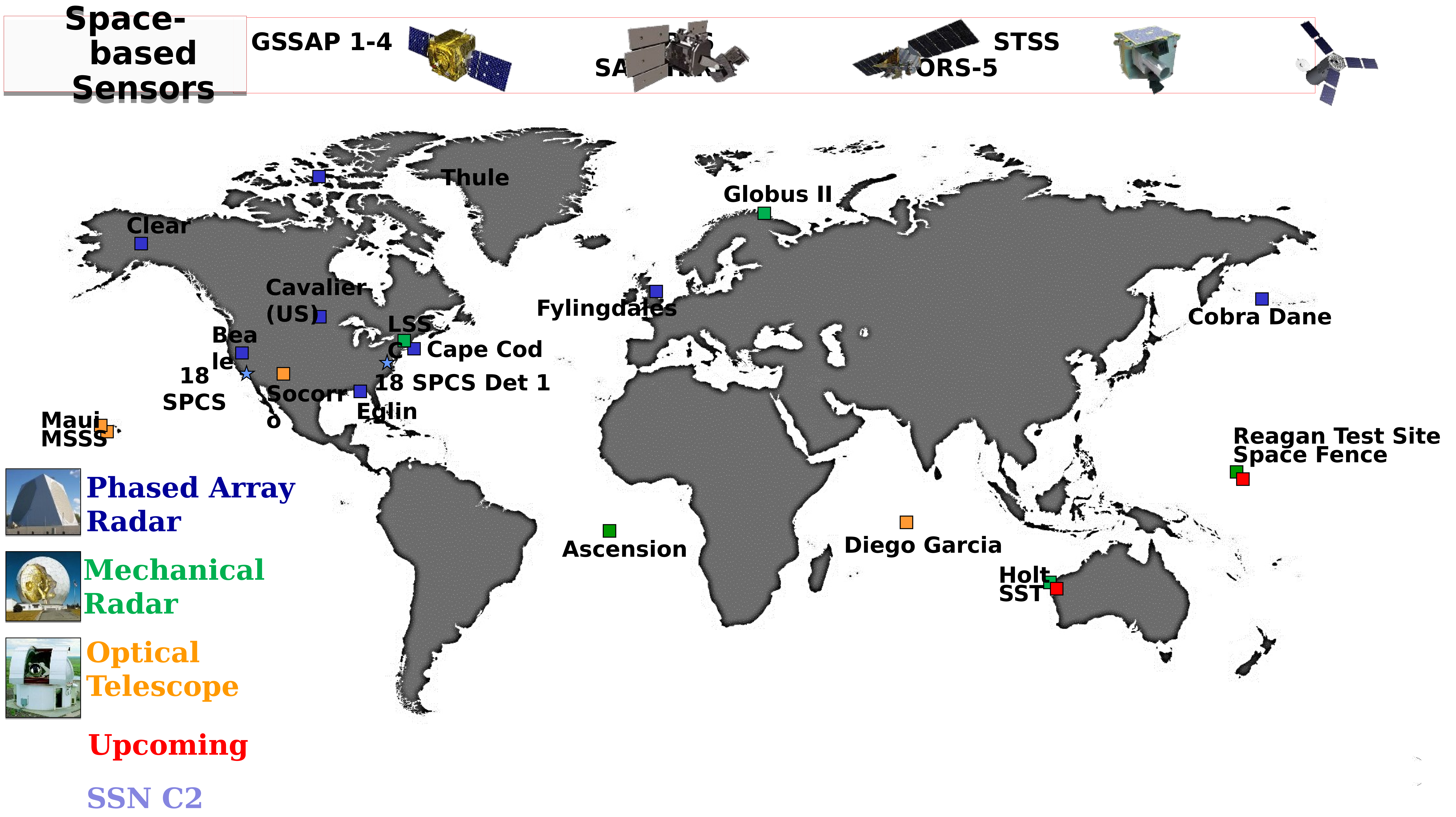
- Probability of Collision (Pc)
  - Statistical measure of the likelihood that the miss distance will be smaller than a specified threshold (usually the combined sizes of the two satellites)
  - Pc calculation requires covariance data (*i.e.,* uncertainty data) on each object’s position estimate; will be discussed later
  - Pc values usually expressed in scientific notation, e.g., 1E-05
    - Large values are 1E-04 and higher
    - Small values are perhaps 1E-06 and lower
- Screening Volume
  - A spherical or ellipsoidal volume around the primary and secondary objects used to determine if a satellite pair is a conjunction candidate
- Collision on Launch Assessments (COLA)
  - Screening performed on powered flight launch trajectory
  - Some entities use “COLA” to mean collision avoidance, or implementation of a risk mitigating action such as a maneuver; this is not standard nomenclature

The Catalog Maintenance Cycle

- Cycle in use since the late 50's, in many forms
- Sensors collect observations and send them to 18th Space Defense Squadron (18 SDS)
- 18 SDS associates submitted observations to objects
- Orbits are updated using observations
- Tasking informs sensors of amount of tracking data needed to maintain desired orbital accuracy



Space Surveillance Network



Observation Types

- Radars typically provide three observables
  - Range to target (the most useful of the measurements)
  - Two angles to target, typically azimuth and elevation
  - Sometimes higher derivatives reported, although useful only if actually observed rather than just calculated
  - Framework used is *topocentric horizon* coordinates, which rotates with earth
- Optical sensors report only two observables, both angles
  - If azimuth mount (axis normal to earth), then report azimuth and elevation
  - If ra/dec mount (axis points to north star), then report right ascension and declination
    - Inertial system better suited to fixed background of stars
  - Space-based sensors usually provide sensor position along with observation

Sensor Tasking

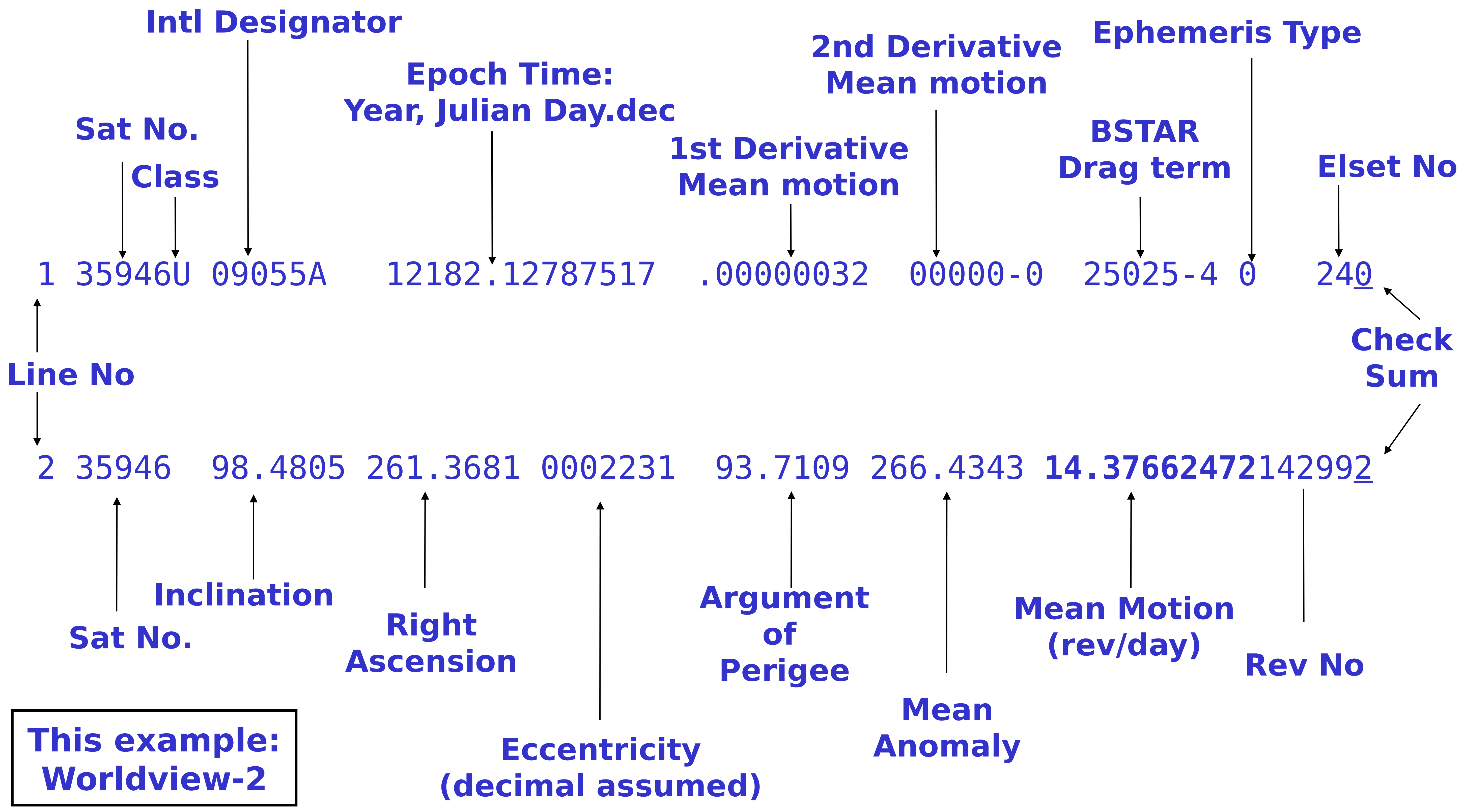
- Sensor capacity is a limited resource
- Tasking function determines collection requirements per object
  - Object type, mission, and orbit determination (OD) solution age determines tasking priority
    - Called tasking "category" with range of values of 1 to 5
  - Number and collection details of tracks required
    - Called tasking "suffix" with large number of alphanumeric codes
- Tasking allocates satellites to sensors
- Establish tracking priority for each satellite
  - Determine sensor/satellite visibility for each satellite/sensor pair
  - Estimate sensor response (detectability) for each pair
  - Specify the number of obs/tracks for each viable satellite/sensor pair
  - "Decentralized execution": sensors told tracking needs/priority for a given day but not precisely when to track
- Composite Tasking List (CTL) sent to all tasked sensors
- Operates on a 24-hour cycle; only one tasking request set per day

Site Mission Planning

- Sites receive the CTL from 18 SDS and plan data collection
- Mission planning allocates limited sensor resources to specific passes
  - Calculate passes using Two-Line Elements (TLEs) from local catalog
  - Estimate sensor response using radar range equation (radars) or visual magnitude (optical)
  - Resource conflicts resolved by tasking category, i.e., when a conflict exists, higher priority satellite pursued
- Observations are collected according to mission plan
  - Plan may be superseded by special tasking in support of Space Situational Awareness (SSA)

Two-Line Orbital Elements (TLEs)

Two-Line Orbital Element (TLE) Format



Will All Tasked Satellites be Tracked? NO!

- Sensor may experience an outage
- Sensor may have bad value for satellite “size” in database
  - Presume cannot be tracked, or allocate too little energy for detection
- Sensor may not have enough energy/capacity to track object
  - Tracking of higher-priority objects took more energy or time than expected
- Position information from 18 SDS may be so poor that satellite not acquired by sensor
- Observation quality may be so poor (large obs covariance) that the track is discarded
- Sensor may mis-assign observations to a different satellite, thus “losing” the tracking information
  - Tracking reported to 18 SDS but may not be associated with proper object in time to be useful

What does all of this have to do with Conjunction Assessment?

- Close approach events become known only when sensors discover the conjuncting objects in the first place
  - Need for wide-area surveillance systems
  - No proposed systems to track down to the 1cm level, which is the hardening level for most spacecraft
- As events develop, additional tracking is desired in order to refine the OD and thus the risk assessment
  - Small objects can be tracked only by certain sensors, so much of SSN capability not helpful for these objects
  - Conjuncting objects are often assigned tasking increases to improve tracking, but this is subject to the vicissitudes of the tasking process

Orbit Determination Concept Description

- Orbit Determination applies a set of force models to a pre-existing orbit estimate and satellite tracking observation to produce an estimate of the orbital state (a “state estimate”) at a particular time (called the epoch time)
- This state estimate can then be propagated forward to estimate the satellite’s position and velocity at a future time
- CA processes involve predicting primary and secondary satellite states forward in time to find the PCA and TCA
  - This process only as good as the underlying dynamical models that produce the epoch and TCA state estimates
  - Thus, some familiarity with OD specifics is necessary to understand CA subtleties

OD Force Modeling: 2-Body Motion

$$\ddot{r} = \boxed{\ddot{r}_{2B}} + \ddot{r}_G + \ddot{r}_D + \ddot{r}_{LS} + \ddot{r}_{RP}$$

Non-Spherical Earth

Atmospheric Drag

Third Body Effects

Solar Radiation Pressure

$$\ddot{\vec{r}}_{2B} = -\frac{\mu \vec{r}}{r^3}$$

where

$\vec{r}$ = Vector from the center of the earth to the object

μ = Gravitational parameter (a constant)

r = Magnitude (length) of the vector

General vs Special Perturbations

- General Perturbations (GP): the theory of TLEs
  - Used for most of the space catalogue for most of SSA history due to computer processing limitations
  - Simplified geopotential (J2-5) and analytic atmospheric drag models
  - Some truncated expressions throughout to simplify calculations
  - No solar radiation pressure modeled
  - Fast but imprecise
- Special Perturbations (SP): the theory of SP vectors
  - All above perturbations represented and handled numerically
  - All integration numeric
  - Relatively slow but quite precise
- TLEs should not be used for CA
  - Not precise enough to drive risk assessment and mitigation
- SP-based products routinely available
  - Provide needed accuracy

Orbit Determination Fit: Sources of Error

- 18 SDS uses minimum variance batch approach to OD fitting
  - Chosen historically because performs well with inaccurate/sparse data
- Most common sources of OD fit error
  - Poor *a priori* sensor observation error weighting
    - Inaccurate sensor observations not de-weighted appropriately,
  - Incorrectly-chosen update interval length
    - Length of look-back period to choose observations for the OD batch update
    - If too long, prediction capability suffers; if too short, drag solution is poor
  - Aliasing between drag and mean motion
    - Both heavily affect in-track position
    - Can be addressed by subdividing drag solution into subdivisions (along-arc solutions)
- OD fit error usually small when tracking is adequate
  - For CA, greater error source is prediction error to TCA

What does all of this have to do with Conjunction Assessment?

- Accuracy of close-approach prediction dependent on quality of OD for primary and secondary objects
  - Primary usually more orbit-stable object and tracked more thoroughly
  - OD quality issues arise more frequently with secondaries
- Problems in modeling of atmospheric drag and solar radiation pressure frequent cause of OD difficulties for CA
  - Solar storms, particularly those that arise in the middle of a CA event, cause particular difficulties
  - Solar radiation pressure is relatively new problem for CA but does influence deep-space CA state estimates and covariances
- If solution is poor, remediation approaches pursued
  - Requests for additional tracking
  - Manual execution of questionable ODs

Orbit Determination Solutions

- Purpose of OD
  - Generate estimate of the object's state at a given time (called the *epoch time*)
  - Generate additional parameters and constructs to allow object's future states to be predicted (accomplished through orbit *propagation*)
  - Generate a statement of the estimation error, both at epoch and for any predicted state (usually accomplished by means of a *covariance matrix*)
- Error types
  - OD approaches (either batch or filter) presume that they solve for all significant systematic errors
  - Remaining solution error is thus presumed to be random (Gaussian) error
  - Sometimes this error can be intentionally inflated to try to improve the fidelity of the error modeling
  - Nonetheless, presumed to be Gaussian in form and unbiased

OD Parameters Generated by ASW Solutions

- Orbit Determination performed by 18 SDS using Astrodynamics Support Workstation (ASW)
- Solved for: State parameters
  - Six parameters needed to determine 3-d state fully
  - Cartesian: three position and three velocity parameters in orthogonal system
  - Element: six orbital elements that describe the geometry of the orbit
- Solved for: Non-conservative force parameters
  - Ballistic coefficient ($C_D A/m$); describes vulnerability of spacecraft state to atmospheric drag
  - Solar radiation pressure (SRP) coefficient ($C_R A/m$); describes vulnerability of spacecraft state to radiation momentum from sun
- Considered: ballistic coefficient (and SRP) consider parameter
  - Not solved for but “considered” as part of the solution
  - Derived from information outside of the OD itself
  - Discussed later

OD Uncertainty Modeling

- Characterizes the overall uncertainty of the OD epoch and/or propagated state
  - Uncertainty of each estimated parameter and their interactions
- This is a characterization of a multivariate statistical distribution
- In general, need the four cumulants to characterize the distribution
  - Mean, variance, skewness, and kurtosis; and their mutual interactions
  - Requires higher-order tensors to do this for a multivariate distribution
- Assumptions about error distribution can simplify situation substantially
  - Presuming the solution is unbiased places the mean error values at zero
  - Presuming the error distribution is Gaussian eliminates the need for the third and fourth cumulants
  - Error distribution can thus be expressed by means of variances of each solved-for component and their cross-correlations
  - Thus, error can be fully represented by means of a covariance matrix

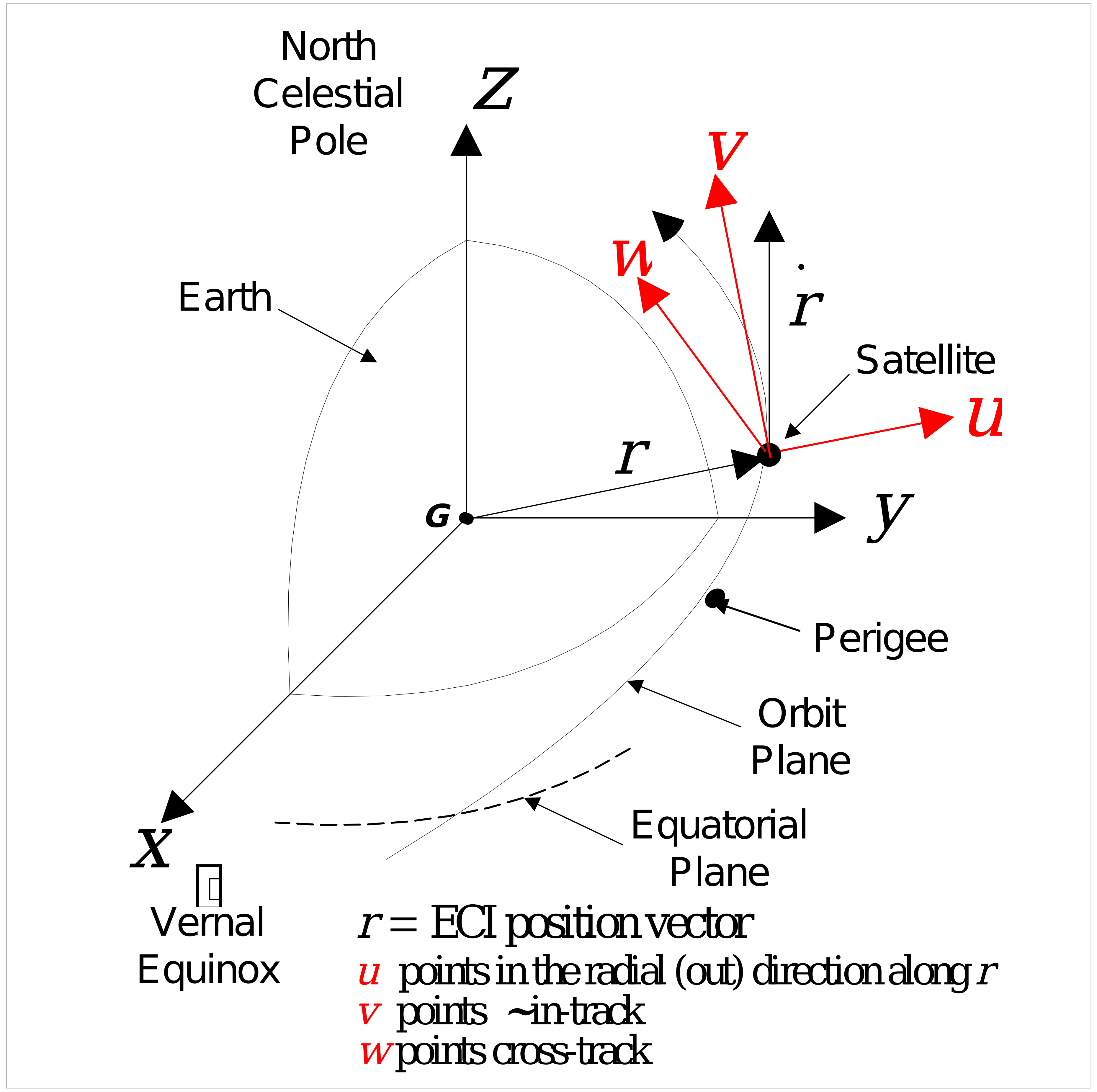
Covariance Matrix Construction: Symbolic Example

- Three estimated parameters (a, b, and c)
- Variances of each along diagonal
- Off-diagonal terms the product of two standard deviations and the correlation coefficient (ρ); matrix is symmetric

	a	**b**	**c**	…
a	σ_a^2	$\rho_{ab}\sigma_a\sigma_b$	$\rho_{ac}\sigma_a\sigma_c$	…
b	$\rho_{ab}\sigma_a\sigma_b$	σ_b^2	$\rho_{bc}\sigma_a\sigma_c$	…
c	$\rho_{ac}\sigma_a\sigma_c$	$\rho_{bc}\sigma_a\sigma_c$	σ_c^2	…
…	…	…	…	…

Covariance often Expressed in Satellite Centered (UVW) Coordinate Frame

- Origin: at satellite
- Fundamental plane: established by the instantaneous position and velocity vectors of the satellite
- Principal direction: along the radius vector to the satellite
- When valid/applicable:
  - Valid at time tag for the point
  - Used to represent miss distances relative to the Primary in an Conjunction Data Message (CDM)
- Unit vectors: u, v, w
  - w is perpendicular to the position and velocity vectors
  - v established by the right-hand rule $w \text{ X } u = v$



Coordinate frame diagram courtesy Omitron Inc.

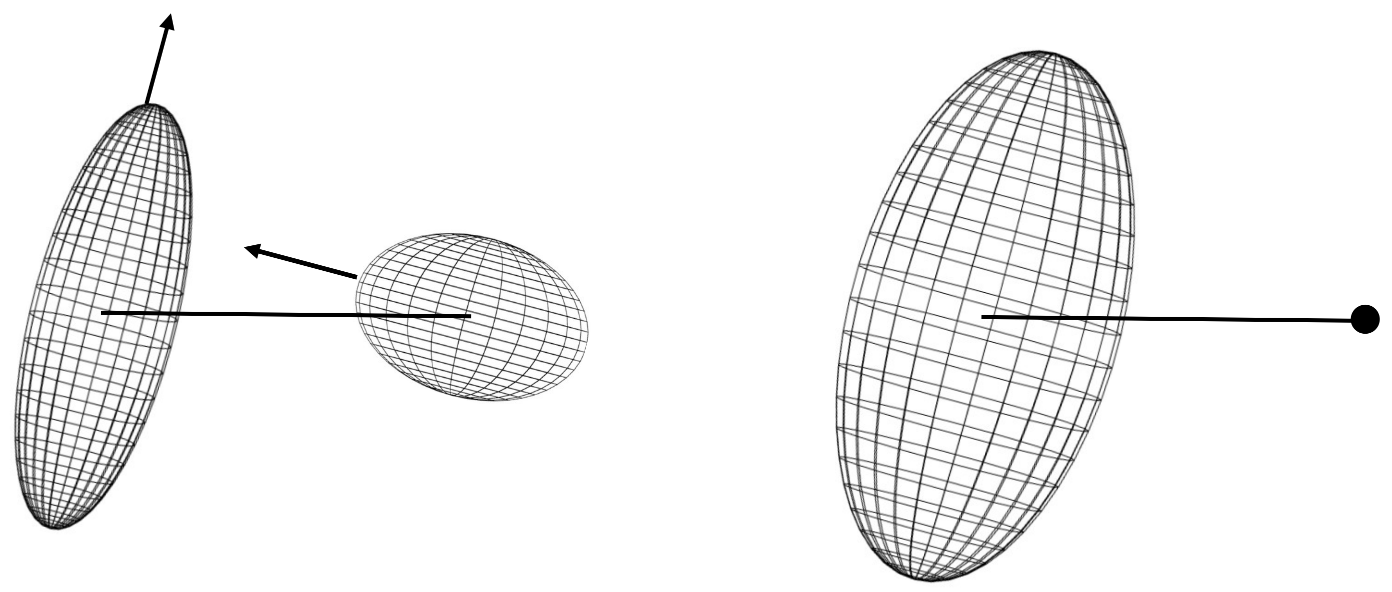
Covariance Propagation Methods

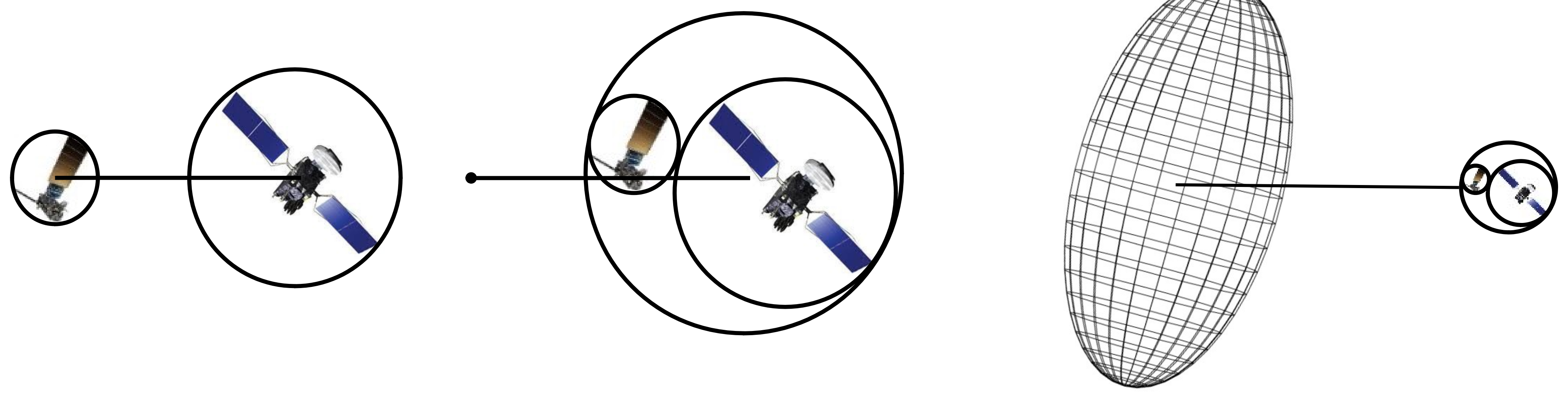
- OD produces covariance at epoch time
- To obtain covariance at TCA, need to employ one of following propagation methods
  - Full Monte Carlo
    - Perturb state at epoch (using covariance), propagate each point forward to t_n with full non-linear dynamics, and summarize distribution at t_n
  - Sigma point propagation
    - Define small number of states to represent covariance statistically, propagate set forward by time-steps, reformulate sigma point set at each time-step, and use sigma point set at t_n to formulate covariance at t_n
  - Linear mapping
    - Create a state-transition matrix by linearization of the dynamics and use it to propagate the covariance to t_n by pre- and post-multiplication
- All three of above methods legitimate
  - List moves from highest to lowest fidelity and computational intensity
  - 18 SDS uses linear mapping approach

Pc Calculation: Methodologies

- Two-dimensional approximation (“2-D Pc”)
  - Most common calculation methodology—fast and straightforward
  - Applicable to most conjunctions
  - Tests and calculations enhancements developed to improve 2-D Pc accuracy and use
- “3-D Pc”
  - Expanded analytical technique proposed by V. Coppola (2012); improved by Hall (2021)
  - Used in situations in which 2-D Pc suspected to miscarry potentially
- Monte Carlo
  - Most accurate methodology to calculate Pc—requires fewest assumptions
  - Also most computationally demanding

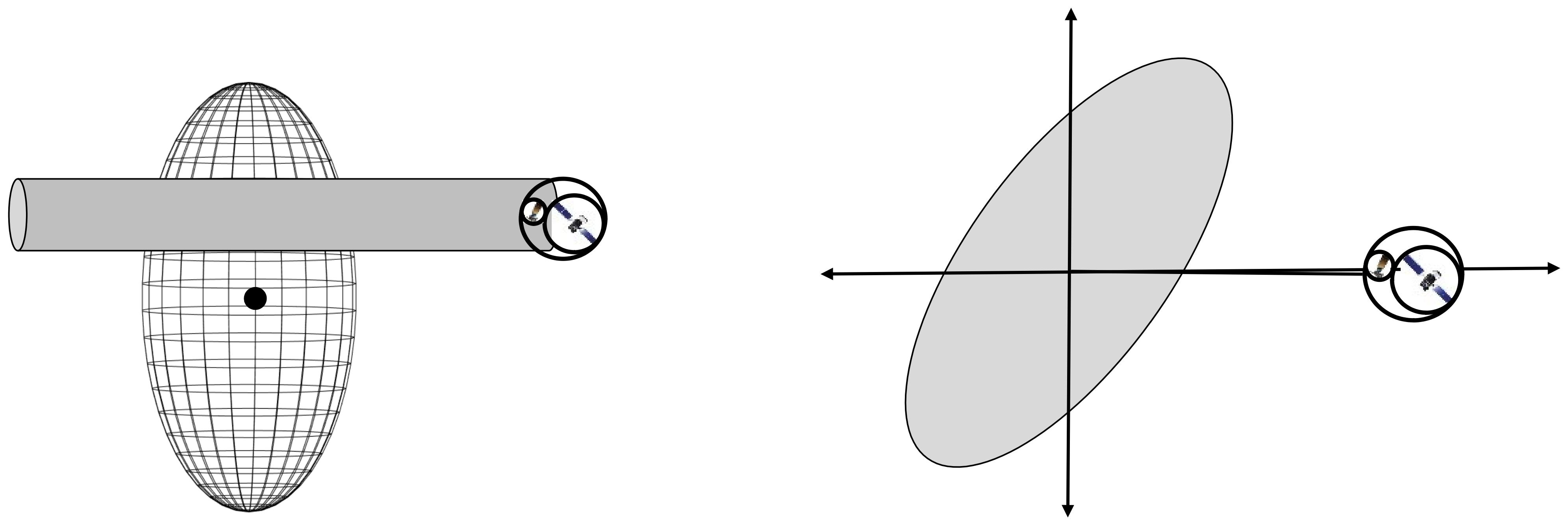
2-D Probability of Collision Calculation





Step I: Primary and Secondary uncertainties combined and placed at position of secondary

Step II: Primary and Secondary object sizes combined with circumscribing sphere and placed at position of primary



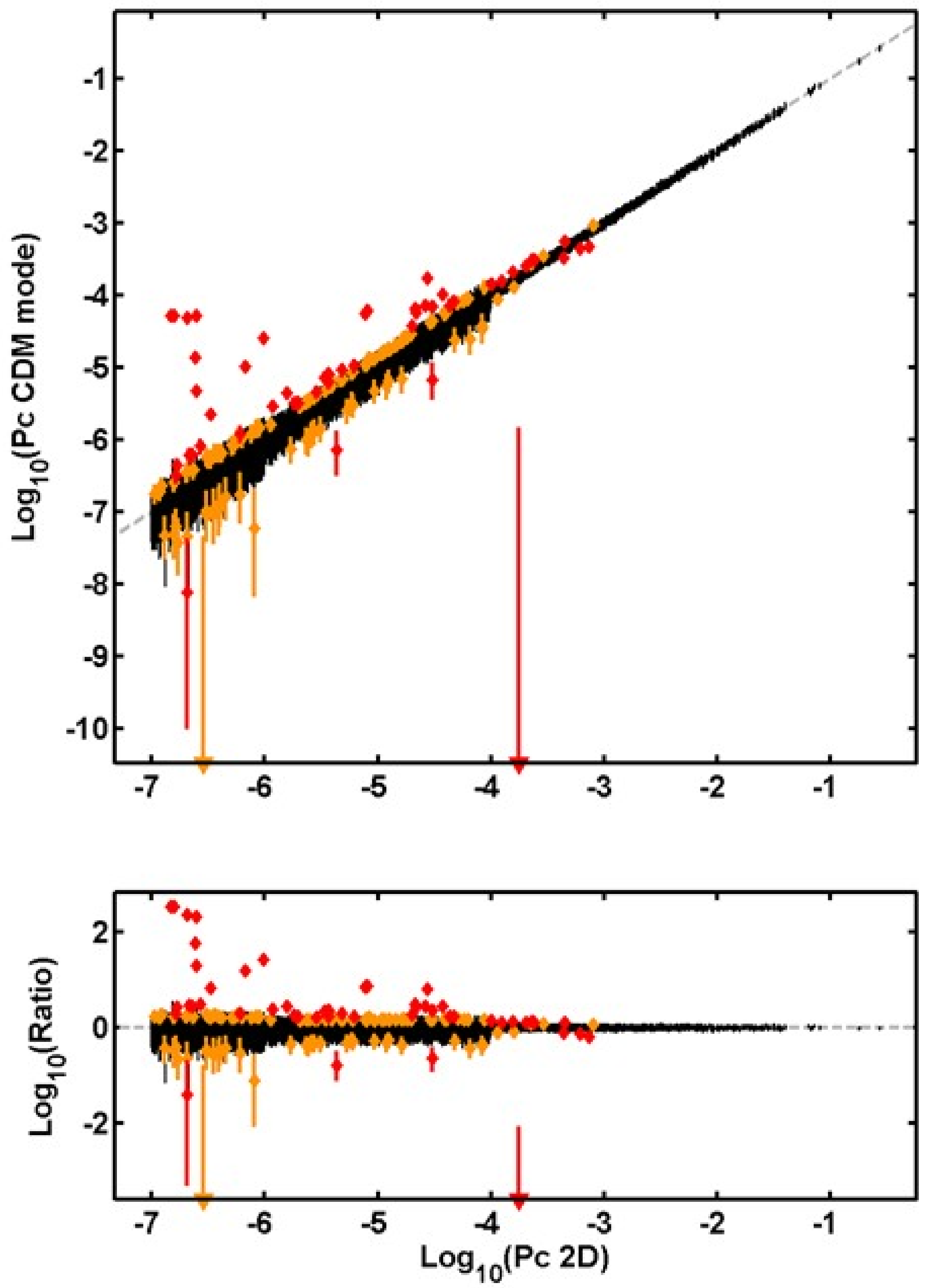
$$P_C = \frac{1}{\sqrt{(2\pi)^2\left|C^*\right|}} \int_A\int \exp\left(-\frac{1}{2}\vec{r}^{\,T} C^{*-1}\vec{r}\right) dXdZ$$

Step III: If collision hyperkinetic, motion approximated as rectilinear. Primary's motion can be considered a straight cylinder, which marginalizes out that component's contribution to probability--can then project situation into plane

Step IV: Probability of collision is portion of covariance probability density that falls within hard-body radius (HBR) circle; as given by above integral

Monte Carlo Pc Calculation: Comparison with 2-D Pc Calculation

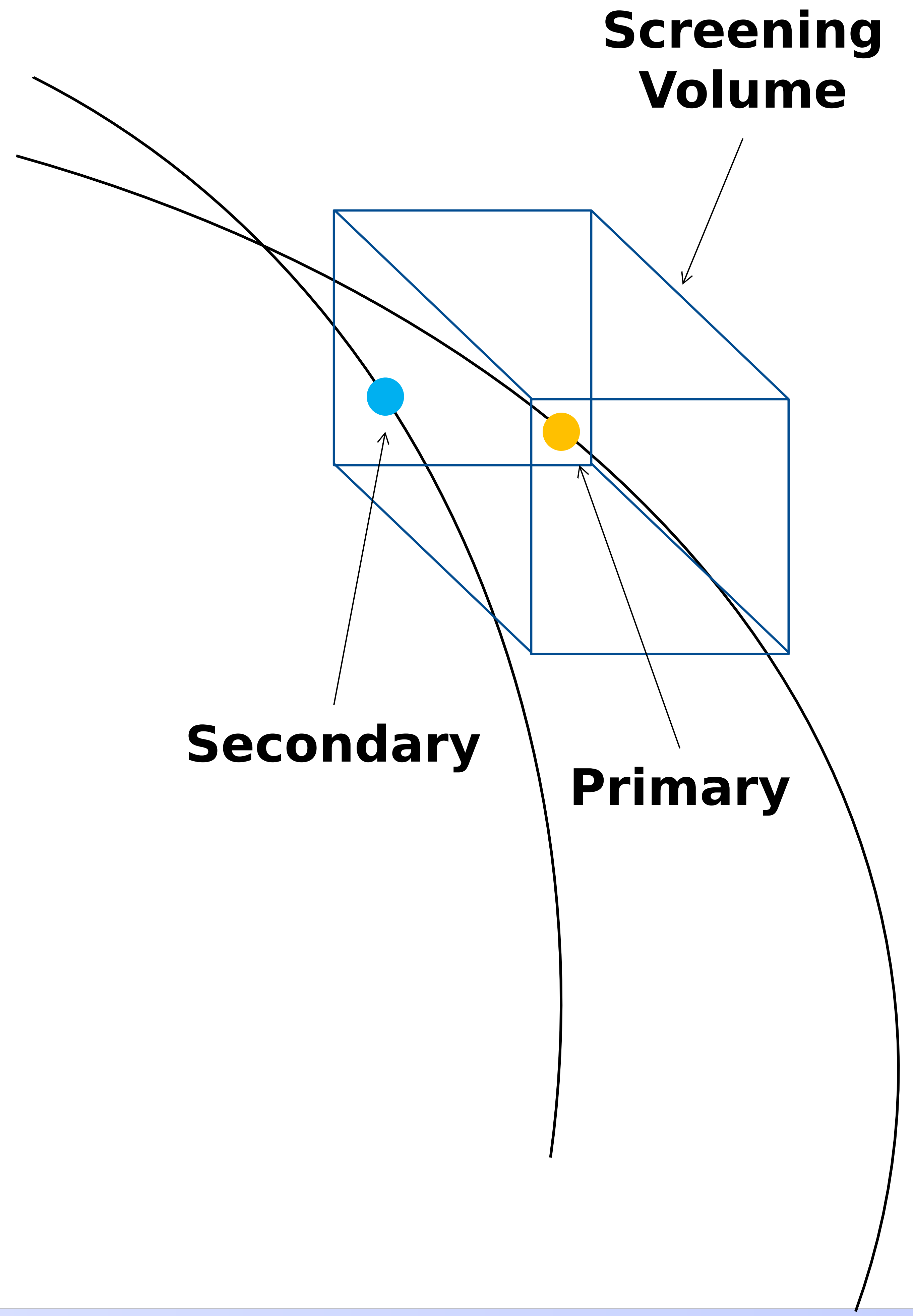
- Analysis compares 28,652 conjunctions from CARA's archive
  - May 2017 to March 2018 with 2D-Pc $\geq 10^{-7}$
- All subjected to a statistical test to find significant Pc differences
  - Null hypothesis: CDM-Pc = 2D-Pc
  - 52 conjunctions have p-value $\leq 10^{-6}$
  - 99 more have $10^{-6} <$ p-value $\leq 10^{-3}$
- Many more major deviations occur than expected from random variations



For a small fraction of temporally-isolated conjunctions, Monte Carlo yields significantly different results than the 2D P_c approximation0

CA Screenings: "Fly By" Ephemeris Comparison

- ✦Generate ephemerides for primary and secondaries that are possible threats
- ✦Construct screening volume box (or ellipsoid) about primary
- ✦"Fly" the box along the primary's ephemeris
- ✦Any penetrations of box constitute possible conjunctions
- ✦For these conjunctions, generate CDM
  - State estimates and covariances at TCA
  - Relative encounter information
  - OD information



CDM Contents: Conjunction (rather than object) Information

- Creation time – not necessarily the time of either OD
  - Time of closest approach (will change slightly with updates)
    - Overall miss distance and relative speed
      - Relative position/velocity in RTN coordinates (another name for RIC or UVW, previously defined)

```
CCSDS_CDM_VERS                    =1.0
CREATION_DATE                     =2015-106T18:19:13.000
ORIGINATOR                        =JSPOC
MESSAGE_FOR                       =                    NASA/GSFC
MESSAGE_ID                        =12345_conj_45678_2015107235948
TCA                               =2015-107T23:59:48.867
MISS_DISTANCE                     =8083                      [m]
RELATIVE_SPEED                    =12067                     [m/s]
RELATIVE_POSITION_R               =-184.5                    [m]
RELATIVE_POSITION_T               =4764.9                    [m]
RELATIVE_POSITION_N               =6526.6                    [m]
RELATIVE_VELOCITY_R               =-21.6                     [m/s]
RELATIVE_VELOCITY_T               =-9745.0                   [m/s]
RELATIVE_VELOCITY_N               =7118.0                    [m/s]
```

Sample Event: Information on Primary

- Basic information on primary object helpful for orientation
  - Usually this information known well, so mostly refresher
  - If O/O ephemeris is used, estimation information not available or useful

CARA - Orbital Information @ TCA

Orbital Parameter	Value
Period (min)	97.6
Perigee Height (km)	681.3
Apogee Height (km)	721.7
Inclination (degrees)	98.2
EDR (W/kg)	2.77e-04
RCS	Large (> 1m^2)

CARA - Estimation Specifics

Parameter	Value
Avg. Tracks Per Day	5.7
Num. Obs. in Span	237
Time of Last Observation	< 24 hours
WRMS	1.55
Ballistic Coefficient (m^2/kg)	0.027
SRP Coefficient (m^2/kg)	0.005

Sample Event: Information on Secondary

- Basic information on primary object needed for evaluation
  - Orbit/size information useful for establishing expected update quality
  - Actual tracking and update information indicates actual quality level achieved

OBJECT - Orbital Information @ TCA

Orbital Parameter	Value
Period (min)	98.7
Perigee Height (km)	674.9
Apogee Height (km)	718.5
Inclination (degrees)	81.2
EDR (W/kg)	5.19e-03
RCS	Small (< 0.1m^2)

OBJECT - Estimation Specifics

Parameter	Value
Avg. Tracks Per Day	0.5
Num. Obs. in Span	15
Time of Last Observation	< 48 hours
WRMS	3.41
Ballistic Coefficient (m^2/kg)	0.682
SRP Coefficient (m^2/kg)	0.285

OBJECT - Safety Volume Violations*

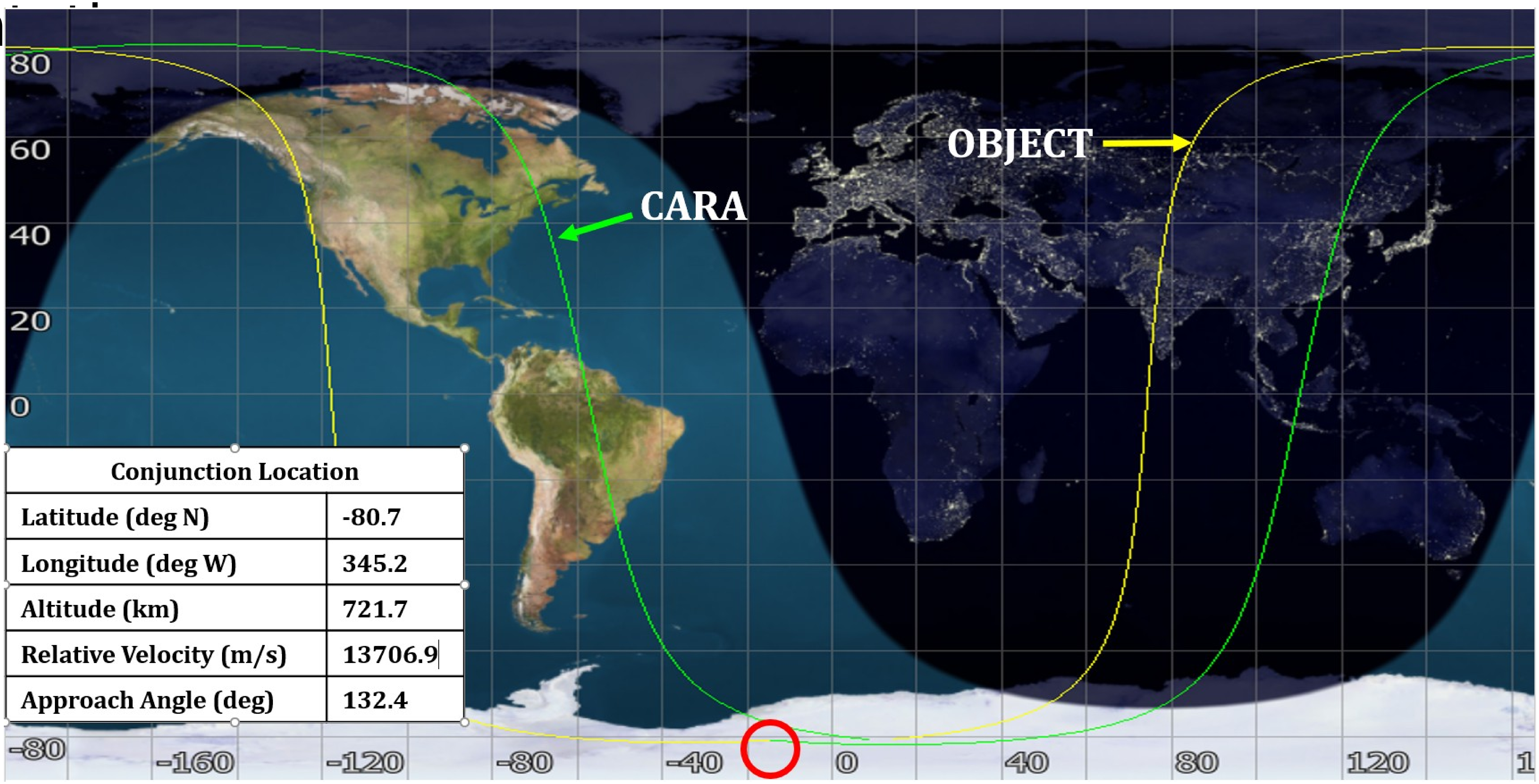
Event Type	Count
# of Predicted Tasking Volume Violations	2
Total # of non-zero Pc events	1

OBJECT - Event Flags

Event Flag	Status
Single Station Tracking	Y
Increased Tasking Requested	Y
Increased Tasking Received	Y
State Update Consistency Check	Y

Sample Event: Ground Trace

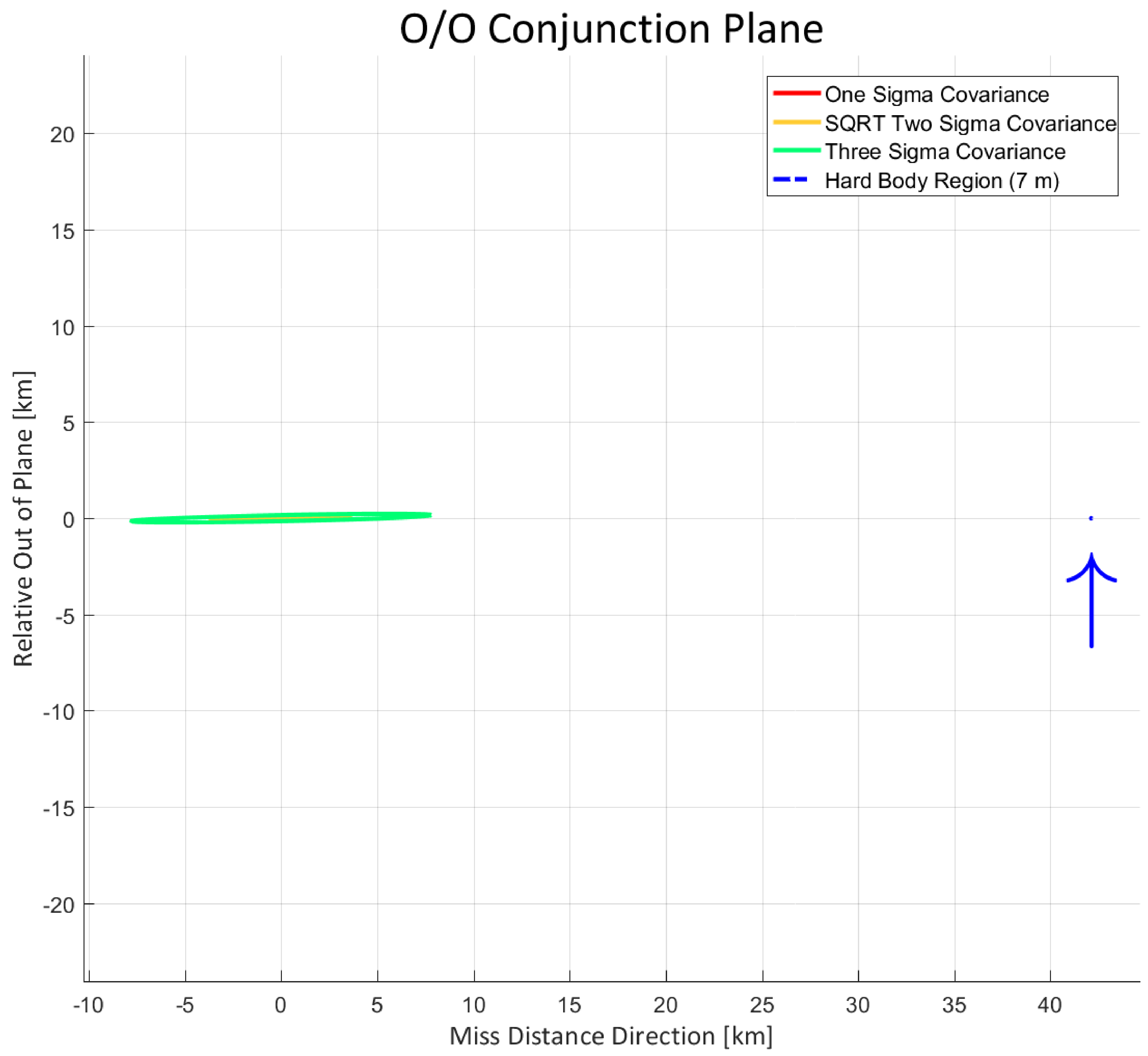
- Not essential for informing mitigation decision, but helpful for orien



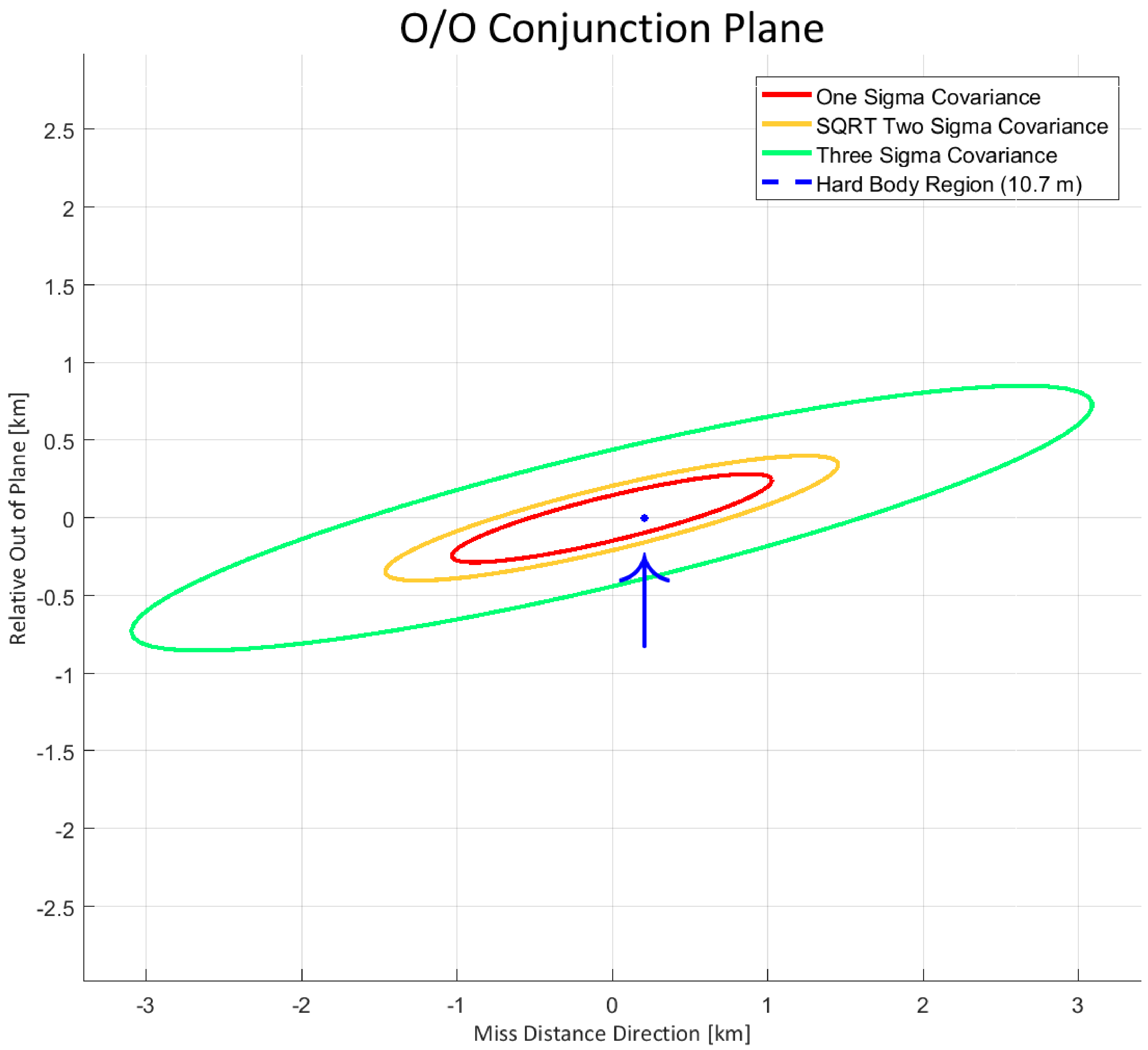
Conjunction Location	
Latitude (deg N)	-80.7
Longitude (deg W)	345.2
Altitude (km)	721.7
Relative Velocity (m/s)	13706.9
Approach Angle (deg)	132.4

Sample Event: Conjunction Plane Plot

- Orients miss distance in context of miss vector uncertainty
- Low-risk situation at left; current event (high risk) at right



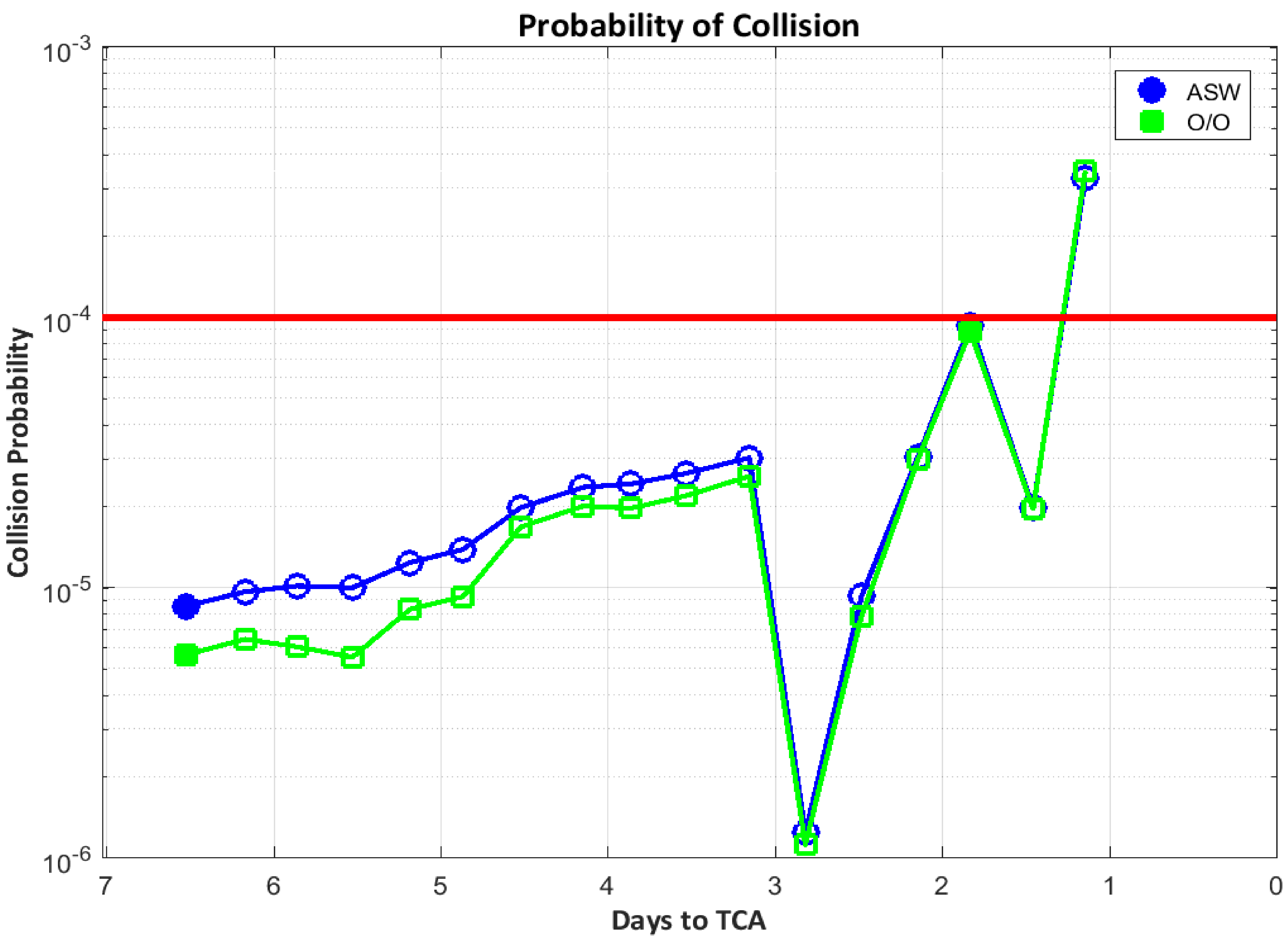
Pc Value : 3.19e-120   Sigma : 23.11   Combined HBR [m] : 7



Pc Value : 3.49e-04   Sigma : 0.38   Combined HBR [m] : 10.7

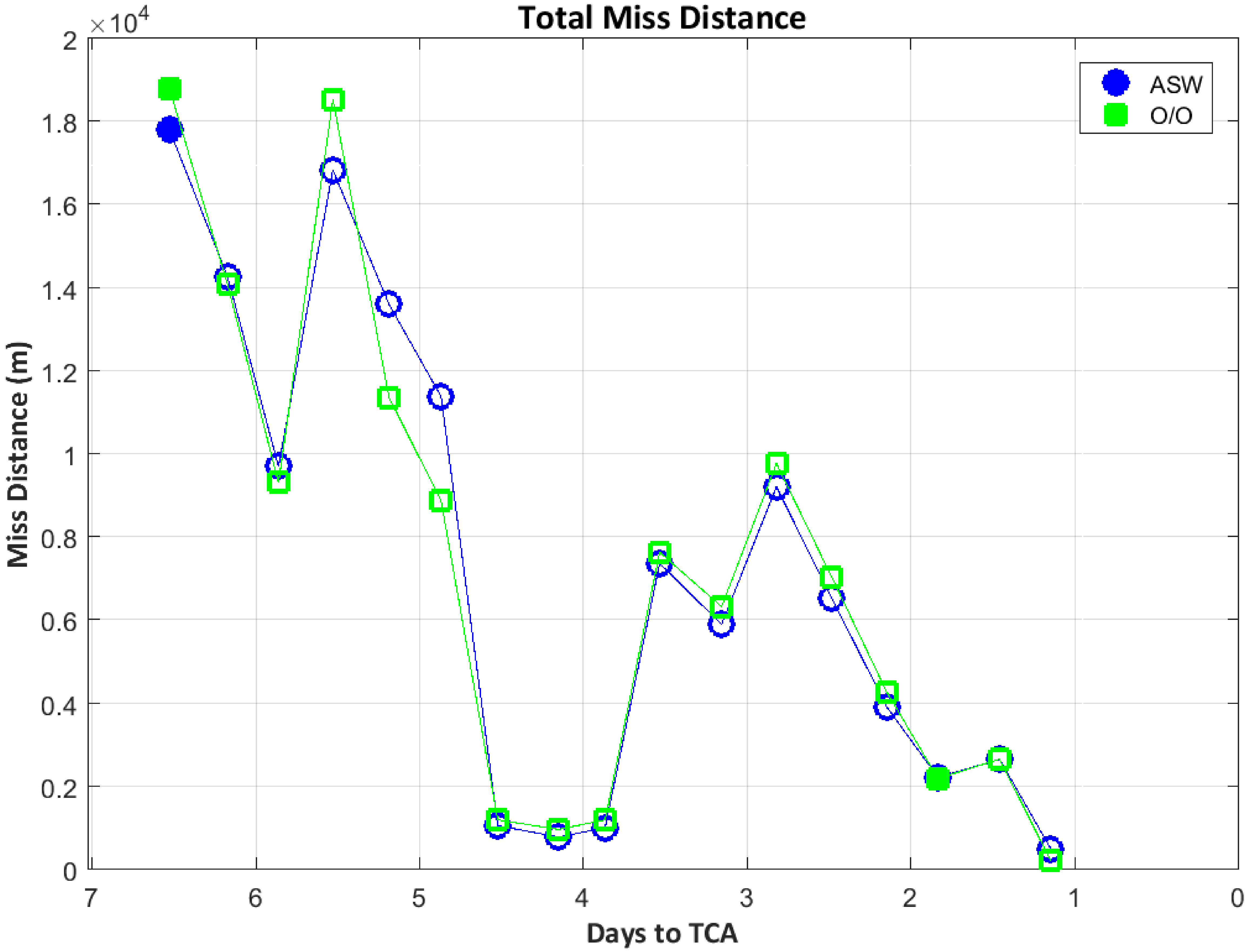
Sample Event: Pc Time History

- Shows behavior of Pc over time, along with tracking information
  - Indicates stab … rability of latest point



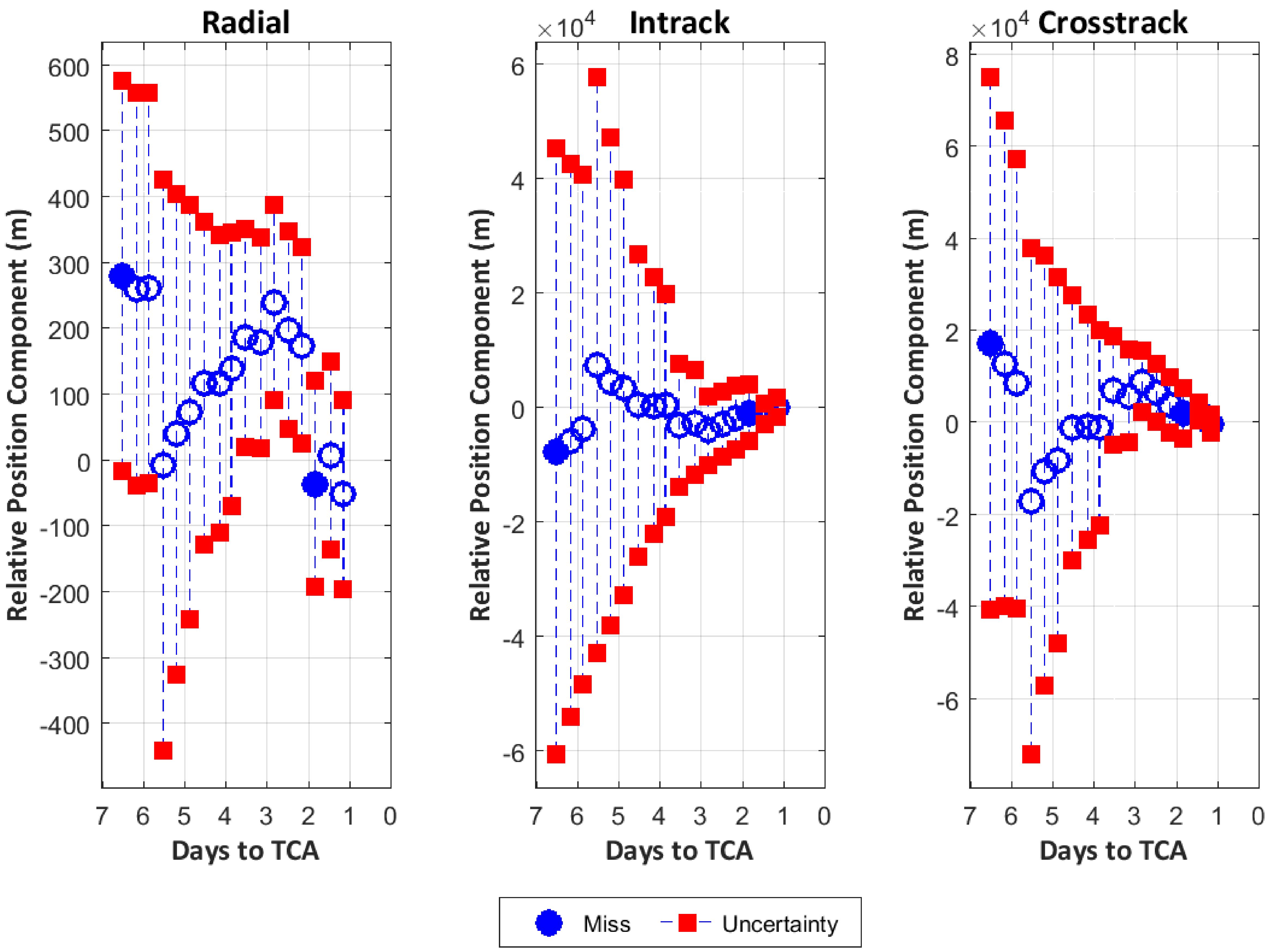
Sample Event: Miss Distance History

- ✦Shows event stability, but little else
  - Componentized miss distance, especially with uncertainty values, more insightful



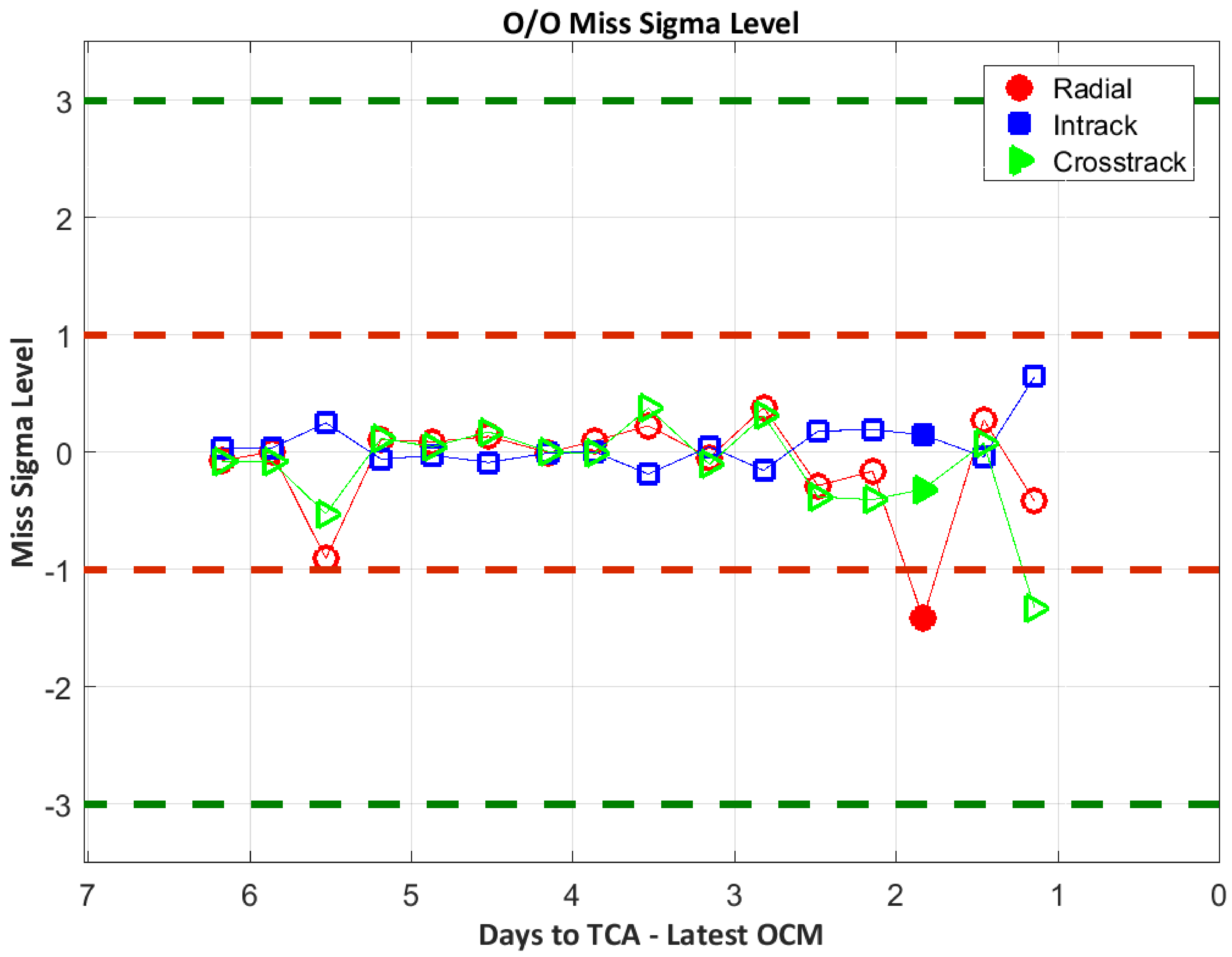
Sample Event: Position Uncertainties I

- Shows event stability in relation to uncertainty
  - Indicates which components unstable and therefore potential source of error



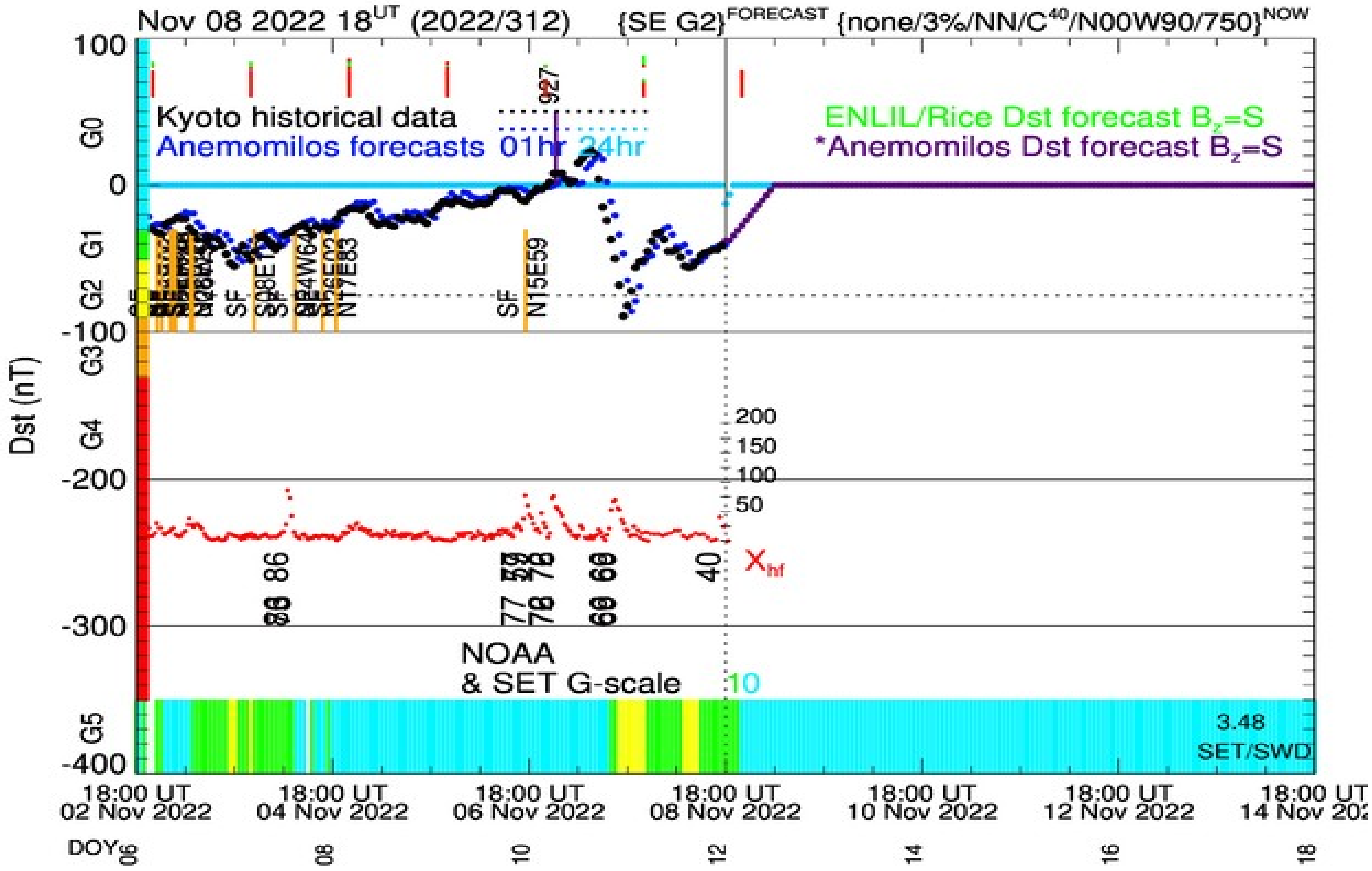
Sample Event: Position Uncertainties II

- Shows event stability in relation to uncertainty, here in relative sense
  - Sigma-level rep … updates more meaningful



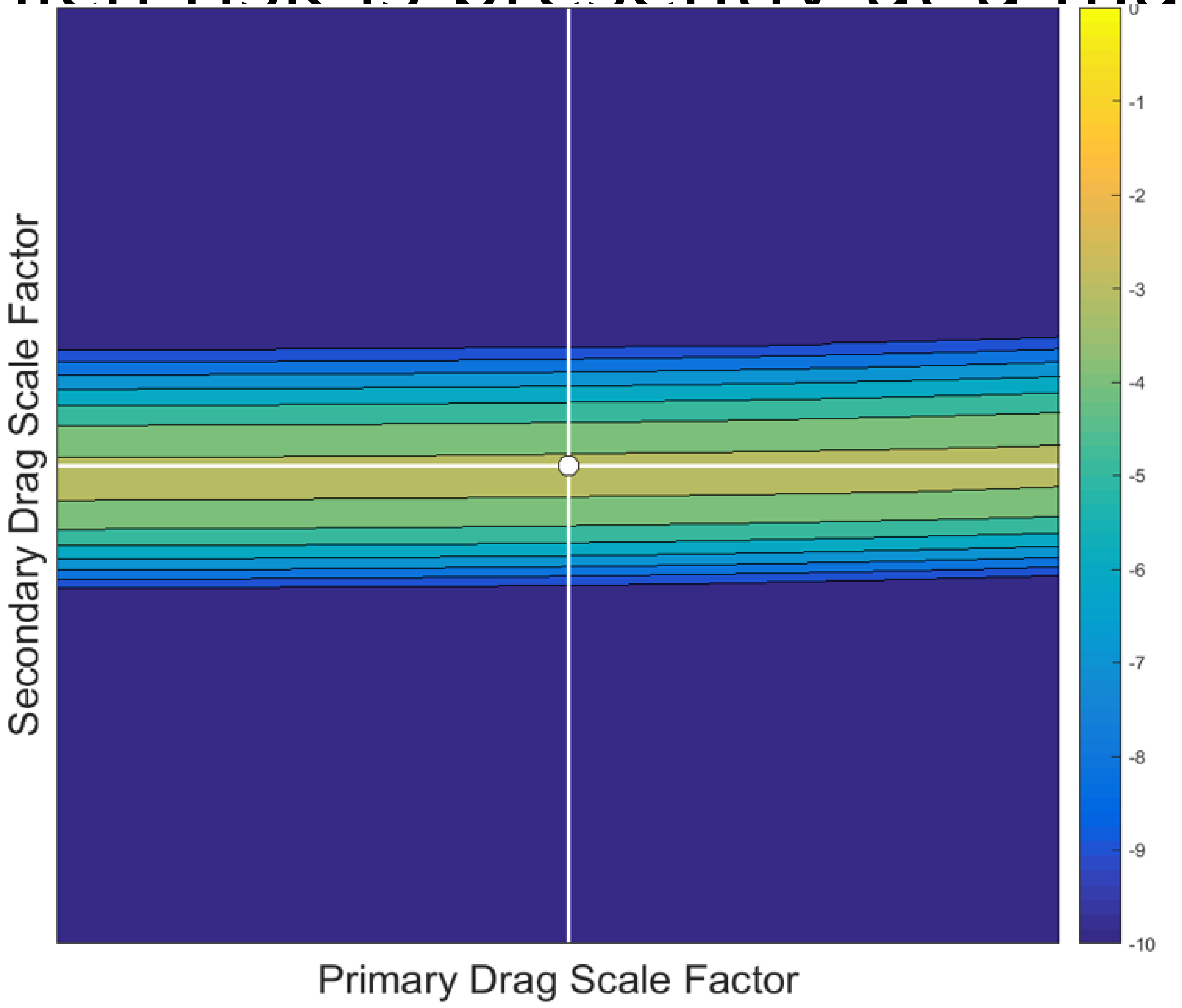
Sample Event: Space Weather Situation

- Can explain past perturbed data and can predict future perturbat



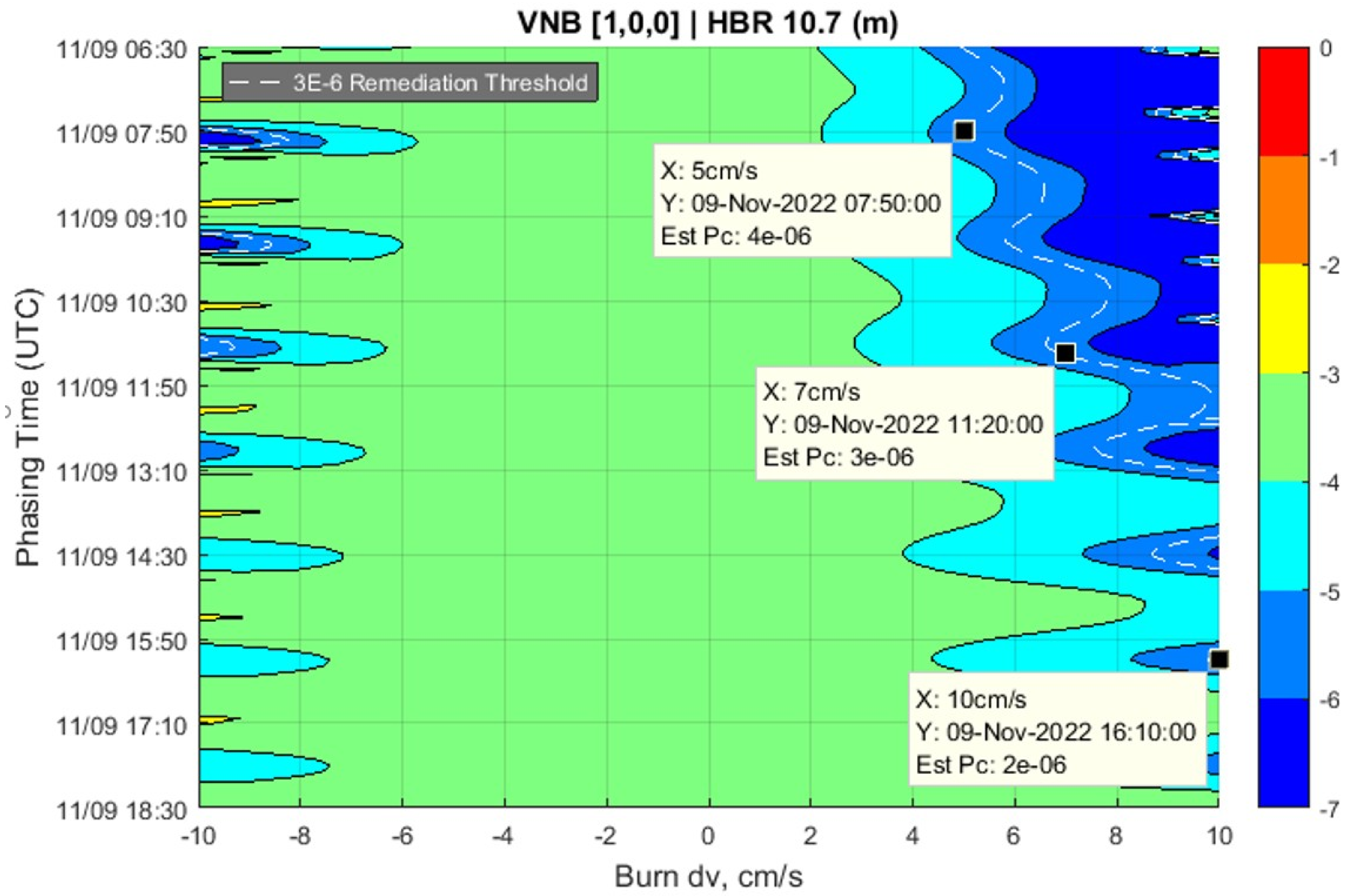
Sample Event: Space Weather Trade-Space

- Determines potential effect on Pc by space weather perturbation
  - Can identify situations in which event not sensitive to space weather perturbation or for which risk is presently at a maximum level and is likely only to decreas



Sample Event: Maneuver Trade-Space

- Identifies which maneuver intensities, at which times, would be required to reduce the Pc to an acceptable level



Sample Event: Summary Slide

Event Summary
CARA vs. OBJECT

TCA:	09-Nov 2022 18:38 UTC
Time to TCA:	1.0 days
ASW OCMs Received:	17
O/O OCMs Received:	17
Last OCM Received:	08-Nov 2022 15:15 UTC
Next Delivery:	09-Nov 2022 00:00 UTC

Current Risk Summary

	ASW	O/O
Probability of Collision	3.29e-04	3.49e-04
Miss Distance (m)	501.5	210.7
Radial (m)	-34.7	-51.2
In-Track (m)	203.0	81.1
Cross-Track (m)	-457.3	-187.6
HBR (m)	10.70	10.70

The CARA team is confident in our risk assessment analysis because:

- We have confidence in the secondary object's epoch state solution.
- We have confidence in the secondary object's state and state uncertainty predictions.

Recommended Course of Action:

- **Execute screened maneuver if Pc remains elevated by commit point**